# UNIT 2
# ARITHMETIC

**Structures in this unit**

expressing arithmetic operations
expressing powers and radicals
using prepositions and past participles in arithmetic operations
explaining arithmetic answers
types of arithmetic questions
greater than / less than / equal to

**Vocabulary in this unit**

carry, simplify, calculate, evaluate, find, round, solve, convert, determine, invert, reduce, expand
multiple / undivided
raise to a power, the power of, squared / cubed, root, radical,
working out

## PREPARATION

**Make a list of all the times you use arithmetic. Do you agree that it is essential? What would your life be like without it?**

### THE PEOPLE WHO DON'T NEED ARITHMETIC

In 1977 a man called Daniel Everett went to the Amazon jungle to meet a tribe of people called the Pirahã.

He learned their language and found that they have a word for 'a lot' and another word for 'a few'. But they have *no* numbers in their language.

Daniel Everett shows how 2 + 2 = 4. The Pirahã people cannot see why they need to know this.

150 acres of Amazon forest are cut down every minute.
150 x 60 = 9,000 acres per hour.

# EXPRESSING ARITHMETIC OPERATIONS

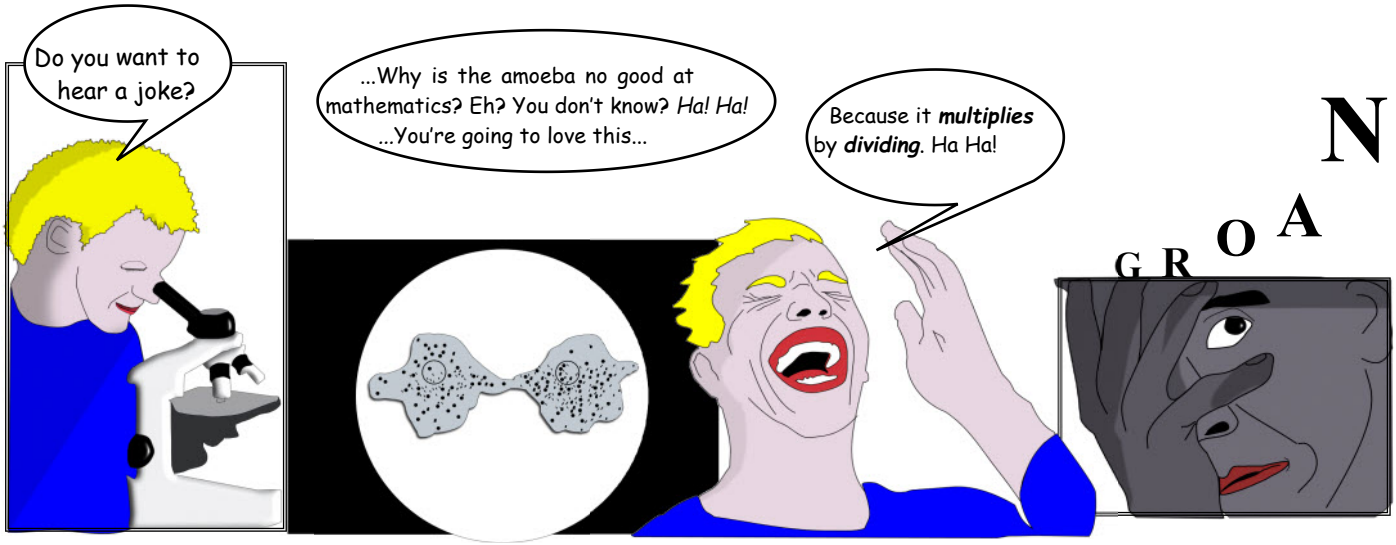
There are several ways to say Addition, Subtraction, Multiplication and Division sums.
You probably know some of the examples below.
Make a note of any that are new to you:

**Addition 3 + 4 = 7**
**add** (*Three **add** four equals seven.*)
**and** (*Three **and** four is seven.*)
**expand** (***Expand** three **by** four to get seven.*)
**increase** (***Increase** three **by** four to make seven.*)
**plus** (*Three **plus** four totals seven.*)
**raise** (***Raise** three by four to get seven.*)
**the sum** (***The sum of** three and four is seven.*)

**Subtraction 9 – 2 = 7**
**decrease** (***Decrease** nine by seven to get two.)*
**deduct** (***Deduct** seven from nine to make two.)*
**difference** (*The **difference** between nine and two is seven.*)
**from** (*Two **from** nine is seven.*)
**less (**two **less than nine** is equal to seven**)**
**minus** (*Nine **minus** two equals seven*)
**subtract** (***Subtract** two **from** nine to get seven)*
**take** (***Take** two **from** nine to get seven.)*
**take away** (*Nine **take away** two is seven.*)
**less** (Nine **less** two is seven)

**Multiplication 6 x 8 = 48**
**multiply** *(**Multiply** six **by** eight to make forty-eight.)*
**times** (*Six **times** eight gives a total of forty-eight.)*
**product** (*The **product of** six and eight is forty-eight)*
*(Forty-eight is the **product of** six and eight*)

**Division 10 ÷ 5 = 2**
**divide** (***Divide** ten by five to make two.*)
**into** (*Five **into** ten goes twice.*)
**share** (***Share** ten between five to make two.*)
**over** (Ten **over** five)



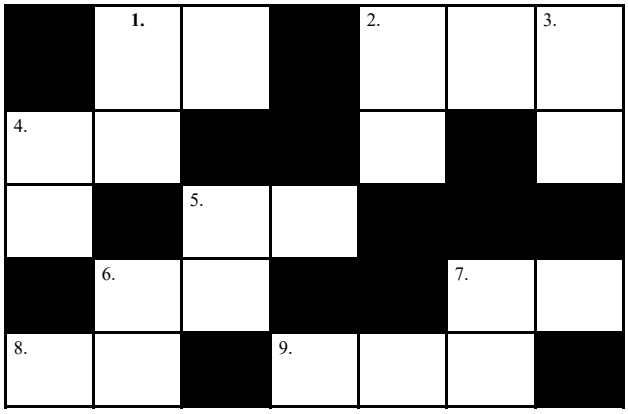
## Exercise 2.1 Name the operations.

**1.** The sum of 2 and 20 is 22. *Addition*
**2.** Take 25 from 40 to make 15. _____
**3.** Raise 15 by 20 to make 35. _____
**4.** 3 minus 2 gives 1. _____
**5.** 3 subtract 2 makes 1. _____
**6**. The product of 6 and 20 is 120. _____
**7.** 2 goes into 6 three times _____
**8.** 6 shared by 2 is 3 _____
**9.** 4 goes into 8 twice _____
**10.** Expand 2 by 8 to make 10. _____
**11.** 2 times 3 is 6 _____
**12**. 2 multiplied by 3 gives the product 6 _____
**13.** 6 shared by 2 gives a quotient of 3 _____
**14.** 3 take away 2 is 1. _____
**15.** 7 less 2 equals 5 ____
**16.** Work out the difference between 5 and 20. _____
**17**. Increase 4 by 20. _____
**18.** Compute 6 plus 20. _____
**19.** Calculate 7 times 20. _____
**20.** What's 20 into 70? _____
**21.** Partition the area into five even sizes _____
**22.** What does 6 times 2 yield? _____

## Exercise 2.2 Use the clues in the boxes to do the puzzle.

**ACROSS**
**1:** The **sum of** 6 and 4
**2:** 150 **take away** 38
**4:** 2 **times** 12 **plus** 5
**5:** 76 **divided by** 2
**6:** **the product** of 25 and 2 **added** to 7
**7:** 96 **halved**
**8:** 100 **less** 10
**9:** 700 **subtract** 76

**DOWN**
**1:** 20 **minus** 1
**2:** 9 **from** 20
**3:** the **difference between** 130 and 101
**4:** **share** 200 **by** 10
**5:** 2 **into** 74
**6:** 100 **decreased by** 50
**7:** 2 **increased by** 42

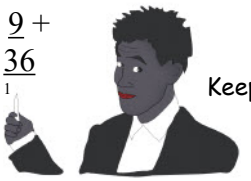



## MORE ARITHMETIC VERBS

To ***reduce*** an amount is to make it one unit of measurement. Example: You **reduce** 68 kilos 15 grams to 68,150 grams. The opposite is **expand**. Example: You **expand** 3312 centimetres to 33 metres 12 centimetres.

You **carry** when digits add up to more than 9.

$$\begin{array}{r} 27 \\ \underline{9} + \\ \underline{36} \\ 1 \end{array}$$



Keep calm and **carry** one.

You **simplify** a fraction by **cancelling** numbers on the top and bottom. **Example**: $\frac{\cancel{28}}{\cancel{32}} = \frac{7}{8}$

You **convert one type of unit** into different units.
Example: You **convert** euros to dollars, miles to kilometres etc.

You **round** a number when you make it simpler. Example: **Round** 703 to the nearest ten to make 700.
9.569 **rounded** to two decimal places is 4.57.

You invert a number by turning it upside-down.

$$\frac{2}{3} \div \frac{1}{4} \rightarrow\rightarrow \frac{2}{3} \times \frac{4}{1}$$

You need to turn some things upside down to be right side up.

## Exercise 2.3 Name these operations:

**1.** 5.5 kilometres → 5,500 metres *reducing*
**2.** 1200 millilitres →1 litre 200ml _____
**3.** $50 → € 47 _____
**4.** $\frac{5}{6} \rightarrow \frac{6}{5}$ _____
**5.** 5.09 → 5.1 _____
**6.** $\frac{11}{33} \rightarrow \frac{1}{3}$ _____

## POWERS and RADICALS

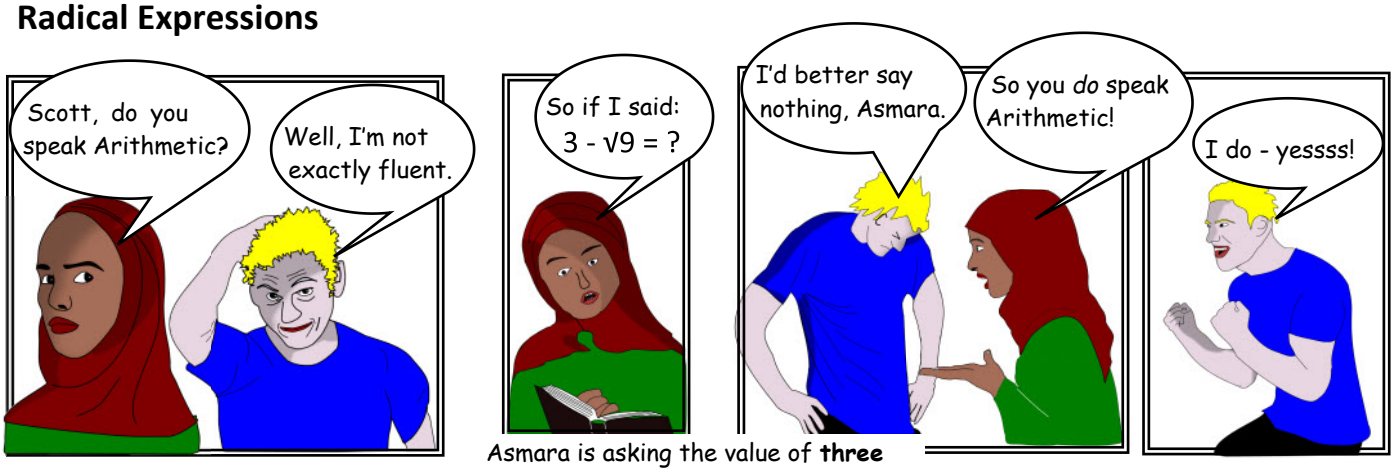
**1. RAISING TO A POWER**

$9^2$ is: nine **squared**
$9^3$ is: nine **cubed**.
$9^4$ is: nine **raised to the power of** four
or nine **to the power of** four.
$9^{1000}$ is: nine **raised to the hundredth** power.
$9^{½}$ is nine to the power of one half

**2. UNDOING A POWER**

**roots** (or **radicals**) undo powers.
√ is the **radical sign**.
√9 is: **the square root of nine**, or root nine or radical nine
$^3$√ is: a **cube root**, or **third root**, or **root three**
$^5$√ is the **fifth root**, or **root five** or **root of five**

## Radical Expressions




Asmara is asking the value of **three minus the square root of nine.'**
The answer, of course, is 0.

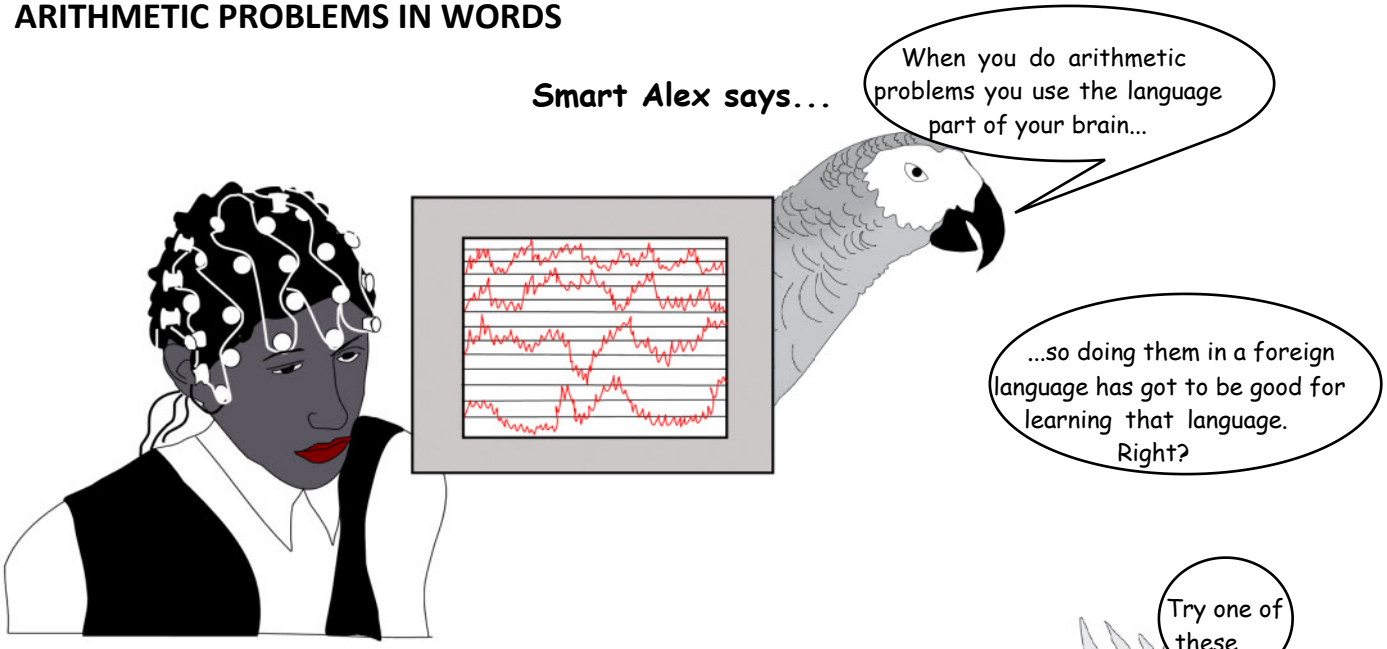
**EXERCISE 2.4 Work with a partner and speak these expressions.**

**Student A**
**1.** $4^3 = 64$ *four cubed equals sixty-four*
**2.** $^3\sqrt{64} = 4$
**3.** $5^{-3} = \frac{1}{5^3}$
**4.** $\sqrt{7} \times \sqrt{2} = \sqrt{14}$
**5.** $\frac{\sqrt{20}}{\sqrt{5}}$
**6.** $(^2/_3)^5$
**7.** $\sqrt{625} = 25$
**8.** $25^{½} = \sqrt{25}$

**Student B**
**1.** $2 \times 4^5$ *two times four to the power of five*
**2.** $\sqrt{-5}$
**3.** $5^3 = 5 \times 5 \times 5$
**5.** $5^{-1} = {}^1/_5$
**6.** $3^6 \div 3^2 = 81$
**7.** $\sqrt{4} = 4^{½}$
**8.** $\frac{6}{\sqrt{5}}$

## ARITHMETIC PROBLEMS IN WORDS

**Smart Alex says...**



**Exercise 2.4**

**1.**
6 cats catch 6 mice in 6 minutes.

How many mice will one cat catch in 60 minutes?

**2.**
Three days ago, yesterday was the day before Sunday.

What day will it be tomorrow?

**3.**
My brother was 18 the day before yesterday. Next year he will be 23.

What's the date of his birthday?

Click here for the answers.
1. 10 mice
2. Thursday
3. 31st December

**When you've done one, explain to a partner *how* you did it.**
**It doesn't matter if you got the answer wrong.**

There are two types of arithmetic problems; the ones you meet in real life and the puzzles designed for maths lessons.
Which type is this one?

The sum of two consecutive integers is 15. Find the numbers.

Yes, it's the second type. No-one outside the strange world of maths, ever talks about 'consecutive integers'.

Here's some advice and some examples to help you understand the language of arithmetic problems.

**ADVICE**

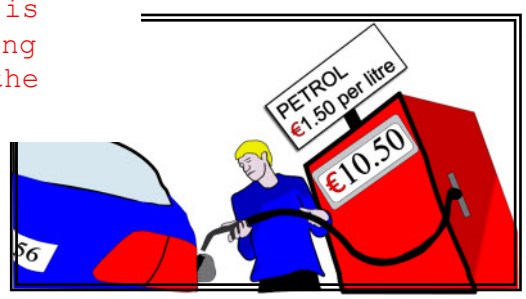
**1.** Read arithmetic problems several times.
**2.** If you can, think about the problems in pictures.
**3.** Look for key words which tell you whether to add, subtract, multiply or divide.
**4.** Words: *find*... *calculate*..., *solve*... and *determine*... all mean 'work out'.
**5.** Make a problem easier to understand by simplifying the sentences or putting the problem in a different way.

**EXAMPLES**

**1.** There were five bags of money here last night. In each bag were €250. A thief has reduced the number of bags by one. Calculate how much money remains.

The word 'reduced' tells you to first subtract one from five.

**2.** Determine the number of litres of petrol a man buys if he is charged € 10.50 at a filling station where the cost of the petrol is €1.50 **a** litre.




Think of the problem in four easy-to-understand sentences:

1. Petrol costs €1.50 per litre.
2. Man goes to buy petrol.
3. He pays €10.50.
4. €10.50 buys how many litres?

## EXPLAINING ANSWERS

Sometimes you have to explain how you solved an arithmetic problem - you have to **show your working out.**

**EXAMPLE 1.**

Five swimmers enter a speed race. Four of the swimmers have had their turns. Their times are 9.8 seconds, 9.75 seconds, 9.79 seconds, and 9.81 seconds. What time must the fifth swimmer get in order to win the competition? *Explain how you get your answer.*

The correct answer to this question is:

*To win, the fifth swimmer must swim faster than 9.75 seconds.*

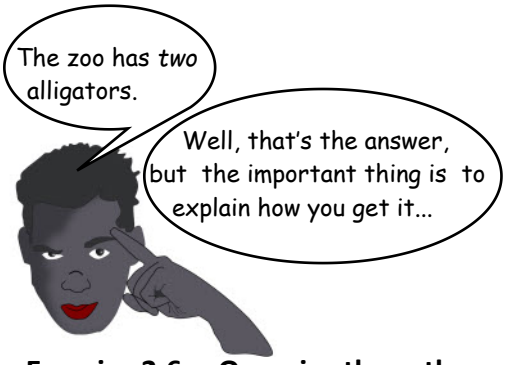
But you have to explain how you get this answer, so ...

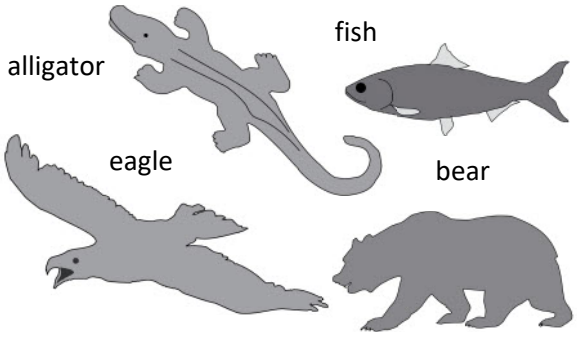
**Exercise 2.5 Choose the best words to complete the explanation.**

I **determined / found / thought** that the fastest time of the four swimmers is 9.75 seconds.
**Because / In order / Therefore** the fifth swimmer must swim faster than that.

**EXAMPLE 2.**

A zoo has equal numbers of alligators, bears and eagles. Each of which eat three fish per day. By the end of the week one hundred and twenty-six fish will have been **shared equally** between all of them. Find the number of alligators in the zoo and explain how you reached that answer.




**Exercise 2.6 Organise these three sentences correctly to explain how to get the answer.**

**Sentence 1.**

that 18 fish are eaten every day

I determined

there are 7 days in a week so

by dividing 126 by 7

**Sentence 2.**

I calculated that there are 6 animals in the zoo

each animal eats 3 fish per day so

by dividing 18 by 3

**Sentence 3.**

and as there are equal numbers of alligators, bears and eagles

6 is the number of animals in the zoo

there are therefore 2 alligators

## PREPOSITIONS

These are the little words which are often connected to verbs.

**Prepositions (by, to & from) and the arithmetic verbs they are connected to:**

**by**
multiply **by** (multiply 3 *by* 2)
divide **by** (divide 9 *by* 3)
decrease **by** (decrease 7 *by* 4)
raise **by** (raise 5 *by* 6)
increase **by** (increase 6 *by* 3)
expand **by** (expand 3 *by* 4)

**to**
add **to** (add 9 *to* 3)
You can also use *add* with no preposition (*9 add 3* or *add 3 and 9*)
(to be) equal **to** (5 plus 2 is equal *to* 7)
convert **to** (convert dollars *to* pounds)

**from**
subtract **from** (subtract 6 *from* 12)
take away **from** (take 9 away *from* 12)
deduct **from** (deduct 9 *from* 27)

**No preposition**
equal (5 plus 2 equals 7)
leave (4 take away 2 leaves 2)
minus (10 minus 3)
add (5 add 4)
make (2 and 3 makes 5)

## PAST PARTICIPLES

Past participles are adjectives or nouns which are formed from verbs. They often end in -ed.
Examples: multiplied, added, divided, subtracted, deducted, decreased, expanded
These past participles always take a preposition.

You can use them in sentences like this:
4 **multiplied** by 6.
4 **added to** 6.
10 **divided by** 5. (This is the same as *divide 10 by 5*)
4 **subtracted from** 6. (This is *not* the same as *4 subtract 6*.)

### Exercise 2.7 Put the correct preposition (or none at all) into the spaces in these sentences.

**1.** multiply 3 __ 4 to make 12.
**2.** 3 times ____ 4 make 12
**3.** 12 divided ___3 equals 4
**4.** 6 added ___3 is 9
**5.** 2 minus ___1 leaves 1
**6.** 1 subtracted ___2 is equal___ 1

### Exercise 2.8 Put these sums into words.

**1.** 12 – 15 = -3 *Twelve take away fifteen is minus three.*
**2.** $\frac{43 - 11}{2} = 16$
**3.** 20 + 7 = 13½ X 2
**4.** 15 ÷ 5 = 3
**5.** $\frac{271 - 11}{130} = 2$
**6.** 15 + 9 - 3 = 7 x 3

### Exercise 2.9. Use the terms in the box to speak sentences 1-5. The first is done as an example.

| |
|---|
| ≠ not equal to |
| > greater than |
| < less than |
| ≥ greater than or equal to |
| ≤ less than or equal to |

**1.** 6 > 17 *Six is less than seventeen.*
**2.** 31 + 69 ≠ 25 + 76
**3.** 15.25 < 15⅔
**4.** The amount of money in the bank is ≤ €5000
**5.** The number of dogs in the park is ≥ 10.

### Exercise 2.10 Choose the correct words to complete the sentences.

**1.** The kilos are **convert / converted / conversion** from pounds.
**2.** You must **inverted / invert / inversion** the fraction.
**3.** Three **expanded / expand / expansion** by four makes seven.
**4. Increase / increase / increased /** three by four.
**5.** Ten **shared / share / sharing /** by five is two.
**6.** It is a **reduced / reduce / reduction** amount.
**7.** You have **to borrow / borrow / borrowing /** ten from the next column.
**8.** Six **multiplied / multiplying / multiplication** by eight is forty-eight.
**9.** Seventeen **deducted from / deducted / deduct** nine is eight.
**10.** Seven is three **raise / raised / raising** by four.

**Exercise 2.11.** 🎧 **Listen to these sums and write what you hear in numbers and symbols.**

**1.** Twenty nine and five eighths added to one and three quarters.

_____

**2.** One and one ninth multiplied by one half.

_____

**3.** Two hundred and seventy point three over ninety.

_____

**4.** Two thousand and five plus ninety point zero one minus seven.

_____

**5.** A half of twenty three equals eleven and a half.

_____

**6.** The product of seventeen and six point five is a hundred and ten point five.

_____

**Exercise 2.12. Work with a partner and speak aloud these sums. The first one is done for you.**

**1.** 53¾ ÷ 91½

*Fifty three and three quarters divided by ninety one and a half*

**2.** ¼ + ⅓ = $^{7}/_{12}$

_____

**3.** ⅞ x ⅝

_____

**4.** ⅜ ÷ ⅛ = 3

_____

**5.** ⅔ - $^{1}/_{16}$

_____

**6.** 2.5 + 7.08 - .043

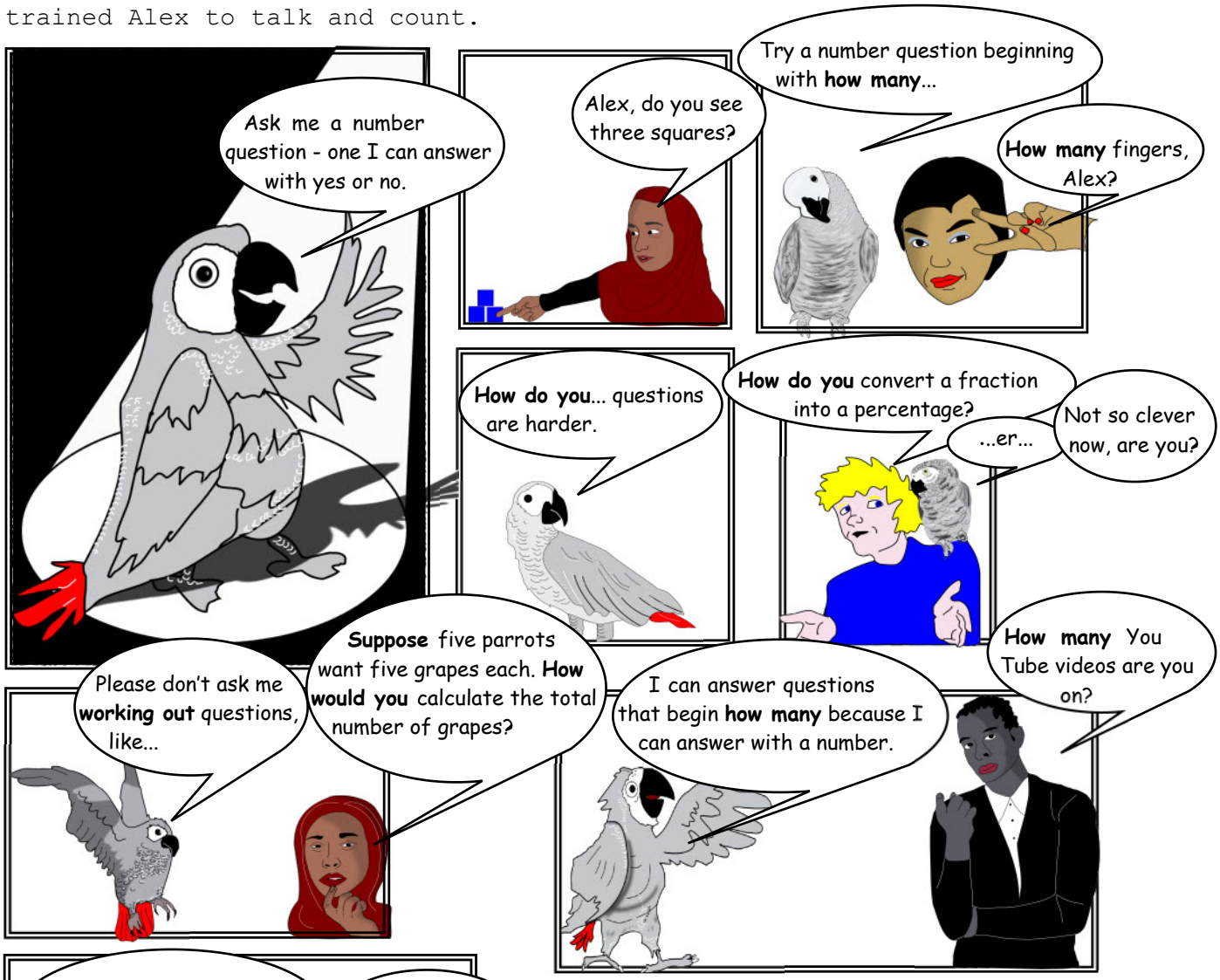
_____

**7.** $\frac{3604}{43.8}$ x $23^{3}/_{4}$

_____

## Types of Questions

Alex is a famous parrot. His name comes from **A**vian **L**anguage **Ex**periment – an experiment which went on for thirty years. Scientist Irene Pepperberg trained Alex to talk and count.



**Exercise 2.13. Put single words in the gaps to complete these questions:**

**1.** _____ number goes into 30, 6 times?
**2.** _____ times does 25 go into 100?
**3.** _____ are there altogether?
**4.** _____ is the total?
**5.** _____ far is it to Paris?
**6.** _____ long does it take?
**7.** _____ less will he have?
**8.** _____ is there altogether?
**9.** _____ are there altogether?
**10.** _____ is the product of the two numbers?
**11.** _____ do you get if you multiply by three?
**12.** _____would you distribute them between five people?
**13.** If we multiply this number by seven, _____ do we get?
**14.** _____do you have to take away to get 50?
**15.** _____do you divide two fractions?
**16.** _____ x = 5, what would the total be?
**17.** _____ 3 the square root of 9?

**FINDING X**
**Episode 2**

Agent Point Double-Oh-Seven boards the plane for Athens

Later at the terminal building...

Finding X continues in Unit 3

**Exercise 2.14. There are things wrong with the grammar of each of the following questions. Correct the mistakes.**

**ADDITION**

**1.** What do that add up to? *What does that add up to?*

**2.** How much are there altogether? _____

**3.** How many is there? _____

**SUBTRACTION**

**4.** How many apples is left? _____

**5.** How many more is needed? _____

**6.** What much is three by nine?_____

**MULTIPLICATION**

**7.** What many will there be? _____

**8.** What are the total? _____